The role of design and training in artifact expertise: The case of mental abacus and visual attention

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Abstract

Previous accounts of how humans develop expertise have focused on how deliberate practice transforms the cognitive and perceptual representations and processes that give rise to expertise. However, the likelihood of developing expertise with a particular tool may also depend on the degree to which that tool fits pre-existing perceptual and cognitive abilities. The present studies explored whether the abacus – a modern descendent of the first human computing devices – evolved to exploit the constraints of human visual attention, or whether, instead, abacus expertise involves extending or adapting the capacity of human visual attention through practice. To address this question, we administered a series of visual search tasks to abacus experts and subjects who had little to no abacus experience, in which search targets and distractors were overlaid atop abacus "beads." Across three studies, we found that experts and naïve subjects were faster to detect targets in semantically-relevant regions of the abacus, suggesting that the attentional biases that scaffold numerical processing in abacus experts require little to no experience with the abacus to develop, and may emerge from general properties of visual attention that are exploited by the design of the abacus itself.

Keywords: visual attention; mental abacus; expertise; artifact design; distributed cognition

One hallmark of human sophistication is our ability to create complex artifacts and to develop expertise in using them. For example, auto mechanics are experts at using specialized tools like car jacks and fender rollers, and neuroscientists are experts at using Magnetic Resonance Imaging (MRI) and Transcranial Magnetic Stimulation (TMS). Previous studies of expertise have explored how attaining expertise might depend on pre-existing individual differences (e.g., Gobet & Ereku, 2007; Smith, Tsimpli & Ouhalla, 1993), but also on how it can be developed through extensive, deliberate practice (Charness, Krampe & Mayr, 1996; Ericsson, Krampe & Tesch-Romer, 1993; Ericsson & Lehman, 1996; Platz et al., 2014; Starkes et al., 1996), and how such practice transforms the cognitive and perceptual representations and processes that give rise to expertise (e.g., Chase & Simon, 1973; Chi, Glaser & Farr, 1988; Chi, Glaser & Rees, 1982; Ericsson & Lehman, 1996; Gauthier, Skudlarski, Gore & Anderson, 2000). However, the likelihood of developing expertise with a particular tool may also depend on the degree to which that tool has been designed to fit the pre-existing perceptual and cognitive abilities of novices. One tool that may exploit general properties of perception and cognition is the *abacus* – a physical artifact that represents exact numerical quantities and arithmetic operations over those quantities via the positions of beads in columns. The fact that the abacus is a descendant of the oldest human computing devices (Ifrah, Harding, Bellos & Wood, 2000; Menninger, 1969) raises the possibility that it may have evolved to become optimally tailored to properties of human perception and cognition. To test this possibility, the present studies explore the effects of artifact design and training on the development of expertise in *mental abacus* (MA): A technique in which users visualize an abacus to perform mental arithmetic.

Understanding how MA expertise develops is important not only as a case study of expertise, but also because it has been increasingly used as a math manipulative in educational settings (see Ball, 1992; Uttal, Scudder, & Deloache, 1997), no doubt inspired by the remarkable ability of MA experts to make calculations rapidly and accurately (Frank & Barner, 2011; Hatano, Miyake & Binks, 1977; Stigler, 1984). However, it is not currently known what drives MA expertise and thus how easily it can be attained in educational settings. If MA expertise depends on perceptual and cognitive abilities that are unusual in the population or that require extensive training, such expertise may be difficult to attain by ordinary children in standard math classrooms. However, if the design of the abacus itself is well-suited to pre-existing perceptual and cognitive abilities, MA expertise may be easier to attain, because it will be scaffolded by the abacus structure.

In the present study, we addressed the effects of design and training on MA expertise by exploring the role of visual attention in MA. MA places heavy demands on visual attention, because encoding the numerical value of an abacus requires attending to a large number of abacus beads. On the one hand, it is possible that through extensive training, expert MA users have learned to attend to the abacus in particularly efficient ways, as has been documented in other case studies of expertise. However, it is also possible that the design of the abacus itself exploits basic attentional biases, leading even novices to attend toward semantically-relevant aspects of the abacus, facilitating their understanding of how the abacus represents number. To test these ideas, we explored how expert MA users and subjects with little to no experience with the abacus attend to abacus-like displays, and thus whether MA practice shapes how attention is allocated over the abacus.

The physical and mental abacus

Children learning MA are first taught how the physical abacus represents number, and are instructed on how to perform basic arithmetic operations like addition and subtraction using highly practiced physical procedures. The abacus represents numerical quantities via specific arrangements of beads organized into columns (see Figure 1). Each column represents a digit with a specific place value, which increases from right to left (i.e., from ones to tens to hundreds, etc.). In the most commonly-used type of abacus, the Japanese *Soroban*, each column is divided by a horizontal beam: below the beam are four *earthly beads*, and above the beam is one *heavenly bead*. When the beads in each column are moved toward the horizontal beam to be *in-play*, they count toward the value of the column, and can be used to represent a digit between 1 and 9. Specifically, when in-play, the heavenly bead in each column counts as a multiple of '5' based on its place value (e.g., 5, 50, 500, etc.) and each of the four earthly beads counts as a multiple of '1' (e.g., 1, 10, 100, etc.). When all of the beads in a column are *out-of-play* – and moved away from the horizontal beam – the column represents 0. Columns representing zero are important for determining cardinality when the zero is *trailing* (e.g., the '0' in 25<u>0</u>) but not when it is *leading* (e.g., <u>0</u>25).

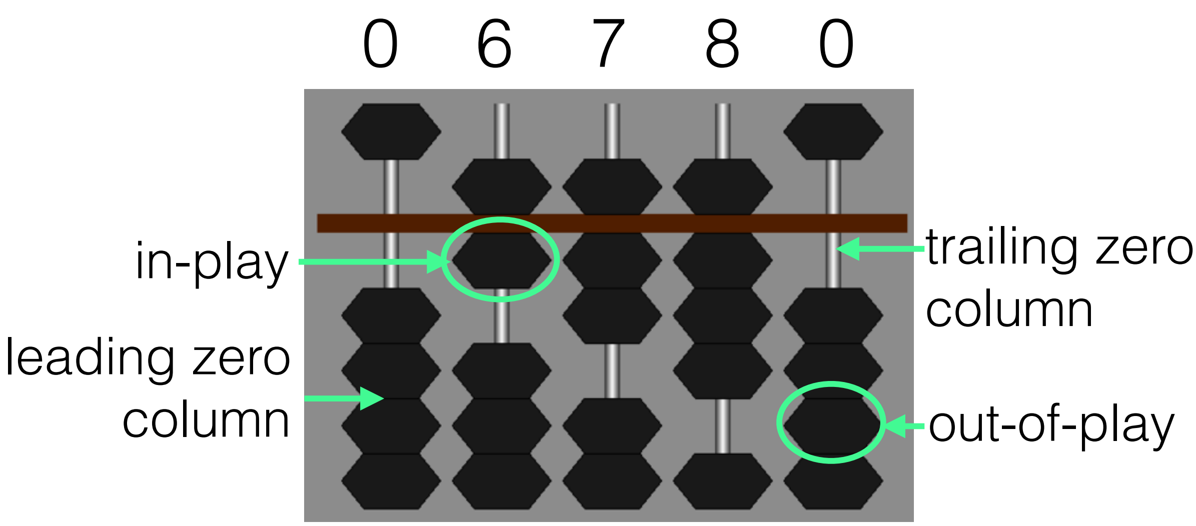

Figure 1. A *Soroban* abacus with highlighted examples of in-play beads, out-of-play beads, and beads in leading zero and trailing zero columns. This abacus represents the number "6,780".

After MA students learn to use the physical abacus, they are trained on mental abacus.

MA students learn to create a mental image of the abacus and to manipulate imagined beads to perform mental arithmetic (Hatano et al., 1977). Experts are able to perform calculations with staggering speed and accuracy (Frank & Barner, 2011; Hatano, Miyake & Binks, 1977; Stigler, 1984). For example, teenage MA users have placed first in the 2010 and 2014 Mental Calculation World Cups, outpacing many older contestants.[1] This expertise is impressive in part because MA requires attending to, holding in memory, and updating the precise locations of a large number of abacus beads in order to represent and manipulate exact numerical values. For example, representing a cardinality like '699' minimally requires representing the locations of 12 beads, and representing them as either in-play or out-of-play, heavenly or earthly, and having a specific place value. The fact that MA users can mentally perform rapid and accurate computations on such numbers – involving numerous abacus beads – is surprising in light of previous studies, which have suggested that there are restrictions on visual attention and working memory. Some studies, for example, indicate that humans can track only 3 or 4 objects simultaneously (e.g., Cowan, 2001; Irwin, 1992; Luck & Vogel, 1997; Todd & Marois, 2004; Vogel, Woodman & Luck, 2001; Vogel & Machizawa, 2004). Others indicate that although there may not be a strict capacity limit on visual working memory, there is nonetheless a negative relationship between the number of items we can store and the precision with which those items are stored (see e.g., Alvarez & Cavanagh, 2004; Bays & Husain, 2008). Thus, by all previous accounts, the ability of MA experts to mentally track the precise locations of numerous beads is unexpected, raising questions about the mental structures and resources required to perform MA.

Although MA appears to exceed previously-reported restrictions on visual working

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[1] Mental Calculation World Cup – the World Championship for Mental Calculators. Retrieved July 15, 2016, from http://www.recordholders.org/en/events/worldcup/index.html

memory, recent evidence suggests that it does rely primarily on visuo-spatial and motor resources. First, several studies suggest that MA stands apart from other forms of mental arithmetic, because it does not depend heavily upon the use of natural language. In particular, while individuals who have not learned MA have great difficulty performing mental computations under verbal interference, MA users are comparatively unaffected by verbal interference tasks, but are instead more impaired by concurrent motor interference tasks (Hatano & Osawa, 1983; Frank & Barner, 2011), which suggests that MA depends more on the motor system than on natural language. Second, and consistent with the above, while standard methods of arithmetic activate cortical areas associated with language and verbal working memory, a large body of work suggests that MA selectively activates regions associated with visuo-spatial and motor processes (Chen et al., 2006; Hanakawa et al., 2003; Hu et al., 2011; Ku et al., 2012; Li et al., 2013a; Li et al., 2013b; Tanaka et al., 2002; Tanaka et al., 2012; Wu et al., 2009).

What is the relationship between MA training and visuo-spatial resources?

Given the fact that MA relies primarily on visuo-spatial resources and that such resources in typical populations appear insufficient for tracking large numbers of abacus beads (Alvarez & Cavanagh, 2004; Bays & Husain, 2008; Cowan, 2001; Irwin, 1992; Luck & Vogel, 1997; Todd & Marois, 2004; Vogel, Woodman & Luck, 2001; Vogel & Machizawa, 2004), one possibility is that attaining MA expertise requires unusually strong visuo-spatial abilities. For instance, those who attain MA expertise might have particularly strong visuo-spatial abilities even before learning MA, which would fit with other examples in which differences in attaining expertise stem from pre-existing, individual differences (e.g., Gobet & Ereku, 2007; Smith, Tsimpli & Ouhalla, 1993). Or, as has been documented in other case studies, MA expertise might require extensive training (Charness, Krampe & Mayr, 1996; Ericsson, Krampe & Tesch-Romer, 1993;

Ericsson & Lehman, 1996; Platz et al., 2014; Starkes et al., 1996) that transforms cognitive and perceptual representations and processes (Chase & Simon, 1973; Chi, Glaser & Farr, 1988; Chi, Glaser & Rees, 1982; Ericsson & Lehman, 1996; Gauthier, Skudlarski, Gore & Anderson, 2000). For example, one possibility is that MA learners develop augmented resources, including a more precise ability to estimate numerosity, and expanded working memory capacities. Consistent with this, some studies suggest that abacus training leads to structural changes in the brain, including enhancements of the white matter tracts that link visuo-spatial and motor areas (Hu et al., 2011; Li et al., 2013a; Li et al., 2013b). These cortical changes may lead to enhanced resources, including more automatic numerical processing (e.g., of Arabic numerals; Wang et al., 2013; Yao et al., 2015).

A third possibility, however, holds that MA training does not lead to augmented visuo-spatial resources, and that MA experts need not be unusually gifted prior to first learning abacus. Instead, MA may be designed to make efficient use of existing resources in typical populations (Frank & Barner, 2011). For example, although MA experts process symbolic numerical stimuli more automatically than naïve subjects who have had no experience using an abacus (Wang et al., 2013; Yao et al., 2015), they are not faster or more accurate at estimating the cardinality of sets of dots (Frank & Barner, 2011; Barner et al., 2016), suggesting that MA experts do not have unusual perceptual expertise. Indeed, both naïve subjects and MA experts are better at numerical estimation when dot arrays are configured similarly to the abacus – e.g., when dots are organized into vertical columns – suggesting that the design of the abacus may be tailored to visuo-spatial processing (Frank & Barner, 2011). Also consistent with the idea that MA training does not lead to generally augmented visuo-spatial resources is evidence that children who are randomly-assigned to learn MA do not develop greater numerical estimation abilities, visuo-spatial working memory capacities, or mental rotation abilities than children who are taught arithmetic using traditional methods (Barner et al., 2016).

Consistent with the idea that MA makes efficient use of existing resources, some computational signatures of MA users adhere to previously-documented limits on visuo-spatial working memory (Alvarez & Cavanagh, 2004; Bays & Husain, 2008; Cowan, 2001; Irwin, 1992; Luck & Vogel, 1997; Todd & Marois, 2004; Vogel, Woodman & Luck, 2001; Vogel & Machizawa, 2004). In particular, when performing arithmetic computations, MA users face difficulty when addends contain more than 3 or 4 digits, i.e., where each addend is represented by 3 or 4 abacus columns. This finding suggests that MA users may be able to track numerous abacus beads (e.g., 12 in the case of ‘699’) because the design and structure of the abacus encourages beads to be chunked into columns, such that each column can be treated a single “object” in visuo-spatial working memory (Frank & Barner, 2011); By this account, MA users may exhibit the limits of visual working memory when abacus columns are considered as units.

In the present study, we took a different approach toward understanding the effects of design and training on MA expertise: Rather than testing whether MA training creates general increases in visuo-spatial resources, we explored whether such training changes *how* visuo-spatial resources are deployed when processing the abacus. Consistent with the idea that practice using the abacus alters the ways in which MA users process the abacus, a recent study using a Stroop-like paradigm found that when MA experts were shown an image of an abacus, they immediately and involuntarily processed its represented number (Du, Yao, Zhang & Chen, 2014). In particular, compared to naïve subjects, MA experts were slower to decide which of two abaci had a larger number of abacus beads when the abacus with the larger number of beads also represented a smaller numeric value (compared to when it represented a larger numeric value; for similar findings with Arabic numerals, see Henik & Tzelgov, 1982).

The finding described above suggests that practice using the abacus leads to automatic processing of its numerical content. But does such practice change how MA users allocate visual attention toward the abacus, or does the design of the abacus exploit basic attentional biases, leading even novices to attend toward semantically-relevant aspects of the abacus? On the one hand, abacus straining could affect how abacus users attend to the abacus: e.g., knowledge that in-play beads (and not out-of-play beads) and beads in columns representing trailing zeroes (but not leading zeroes) affect the number represented by the abacus could lead MA experts to preferentially allocate their attention toward these aspects of abacus structure. This would be fit with previous case studies showing that deliberate practice transforms domain-relevant representations and processes (Chase & Simon, 1973; Chi, Glaser & Farr, 1988; Chi, Glaser & Rees, 1982; Ericsson & Lehman, 1996; Gauthier, Skudlarski, Gore & Anderson, 2000), including the allocation of attention task-relevant items (e.g., McCormack et al., 2014; Sheridan & Reingold, 2007). Naïve subjects, who do not know how the abacus represents number, might therefore fail to exhibit these attentional biases and might instead allocate their attention evenly across the abacus. On the other hand, practice using the abacus need not shape low-level attentional processes. Instead, the design of the abacus – having evolved over many centuries (Ifrah, Harding, Bellos & Wood, 2000; Menninger, 1969) – may exploit attentional biases that are shared by all humans. By harnessing such biases, the structure of the abacus could lead even naïve subjects to preferentially attend to numerically relevant aspects of the abacus, like in-play beads. These attentional biases could facilitate learning how the abacus represents number, and with practice, processing of the abacus' numerical content could become automatic.

What general aspects of visual perception and attention might lead even naïve subjects to direct attention toward semantically-relevant regions of the abacus, like in-play beads? A large literature on selective attention suggests that early visual processes create a *saliency map* of a visual scene (Koch & Ullman, 1987; see also Itti & Koch, 2001; Itti et al., 1998; Li et al., 2002; Treisman & Gelade, 1980), which identifies locations in a scene that are particularly conspicuous relative to surrounding areas, and are thus more likely to attract attention – e.g., due to their contrasting color, size, or orientation. In the context of the abacus, the horizontal beam appears relatively conspicuous, given its distinct thickness and color, and contrasting orientation with the vertical columns of beads (see Figure 1). Critically, if attention is allocated toward the horizontal beam, other items in nearby locations might also be selected. This would predict an overall attentional advantage for in-play beads, which are typically closer to the beam than out-of-play beads. Moreover, because in-play beads within each column are directly adjoined to the horizontal beam and to one another, they could together be treated as a single "object". In light of work suggesting that objects – and not merely spatial locations – can be the foci of attention (for review, see Chen, 2012; Scholl, 2001; see also Duncan, 1984; Egly, Driver & Rafal, 1994; Luck & Vogel, 1997; Treisman, 1982; Vecera & Farah, 1994), this would predict that attention may automatically extend across all in-play beads.

The present studies

In the present studies, we set out to explore whether MA experts exhibit attentional biases toward semantically-relevant aspects of the abacus, and if so, whether these biases emerge from training using the abacus, or if they are instead shared by naïve subjects and exploited by the design of the abacus itself. To measure how attention is allocated to the abacus, we conducted a series of visual search tasks, and presented MA experts and naïve subjects with abacus-like displays in which search targets and distractors were overlaid atop abacus "beads". We reasoned that participants would be faster to detect targets in locations toward which they preferentially attend. Using this method, we asked several questions.

In Experiment 1, we asked how MA experts allocate their attention while reading the abacus. To address this, MA experts were asked to complete a visual search task while simultaneously reporting the number represented by the abacus. Experiment 2 then asked whether the attentional biases exhibited by MA experts in Experiment 1 only emerge when experts are explicitly asked to read the abacus, of if instead these biases may be more automatic. To test this, MA experts were asked to complete only the visual search task, and were not asked to report the number represented by the abacus. Finally, in Experiment 3 we explored whether the ways in which experts allocate attention to the abacus are due to their experience using the abacus, or if instead, the structure of the abacus itself mediates how visual attention is allocated. To test this, we explored whether subjects with little to no experience or knowledge of the abacus preferentially attend to semantically-relevant aspects of abacus structure, by having them perform the visual search task from Experiments 1 and 2.

Experiment 1

The goal of Experiment 1 was to explore how abacus experts allocate their attention toward the abacus when they are reading the number it represents, as an index of which aspects of abacus structure they find most semantically-relevant. To test this, subjects were asked to perform two concurrent tasks. First, they performed a visual conjunction search task (Treisman & Gelade, 1980) in which search targets and distractors were overlaid atop schematic abacus beads. We manipulated whether search targets were located on in-play or out-of-play beads, and whether targets were located on beads in columns representing trailing or leading zeroes. Second, on each trial of the visual search task, subjects were also asked to read and report the number represented by the abacus, which changed from trial to trial. We reasoned that if in-play beads and beads in trailing zero columns are especially relevant to extracting the number represented by the abacus, then when experts are simultaneously reading the abacus, they should preferentially attend to and thus be faster to detect targets at those locations, compared to when targets are on out-of-play beads or on beads in leading zero columns.

Method

Participants. The participants were 63 MA students,[2] who had a mean age of 10.62 years (range: 8.42 to 15.0). All MA participants in this experiment and in Experiment 2 were enrolled in Universal Computation Mental Arithmetic System (UCMAS) franchise schools, located in Gujarat Province, India. MA students were included in our subject pool if they 1) had completed Level 4 UCMAS training (which comprises training using the physical abacus, and an introduction to MA), 2) were judged by their instructor to be among the best students in their class, and 3) could travel to our test site in Vadodara, India. Seventeen of the participants from Experiment 1 also participated in Experiment 2, with experiment order counterbalanced.[3] Some participants also received other measures, reported in Brooks et al. (under review).

Materials and Procedure. Participants completed two concurrent tasks. In the *Visual Search* task, subjects were shown an abacus schematic in which targets and distractors were overlaid atop abacus beads. Participants were asked to find the target item (i.e., a large circle) and report whether it was red or blue, while ignoring distractor items (i.e., large squares and small circles that could be either red or blue). Concurrently, in the *Abacus Reading* task, subjects

[2] We lack demographic information for 10 of these participants

[3] There were no main effects or interactions involving experiment order among the subjects who participated in both Experiments 1 and 2.

were asked to report the number represented by the abacus schematic that was presented on each trial.

Participants were seated in front of Macintosh laptop screens, wore headphones throughout the task, and provided responses using the laptop keyboards and attached USB numeric keypads. Stimuli were presented to subjects using custom software designed using the Psychtoolbox 3 module of MATLAB (Kleiner et al., 2007). Prior to the task, small groups of participants were given instructions by the experimenter in English (the language of instruction at UCMAS), and these instructions were illustrated using examples of several trials of the Visual Search and Abacus Reading tasks. Each participant then completed a brief training phase before beginning the main task. During both training and test phases, feedback for a correct answer was given with a green cartoon smiling face accompanied by a high pitch double tone, and feedback for an incorrect answer was given with a red frowning face accompanied by a low pitch single tone.

Training phase. The training phase included two stages. In the first stage, participants received training on how to indicate the color of the target item during the Visual Search task. Participants first received two trials in which they were shown either a blue or red large circle, in succession, and were asked to press a button on the left or right side of their keyboard that matched the circle in color (e.g., "Please press the "blue" button on the left"; the "blue" button was the 'z' key, marked with a blue sticker, and the red button was the '/' key, marked with a red sticker). After this, participants received an additional six trials in which they were shown a large circle and were asked to press the color-matching button (the circle was red on three trials and blue on three trials). If a participant responded incorrectly on any trial, that trial was repeated. Once all six trials were completed, participants proceeded to the second stage of training.

In the second stage of training, participants were introduced to the Visual Search and Abacus Reading tasks. First, they were shown a schematic of an abacus, which served as the search array. All of the abacus beads in the array except the target bead were overlaid with distractor items. These distractors were randomly-generated, and could be either big squares or small circles, and red or blue. The target bead was overlaid with a big circle, which was also randomly assigned to be either red or blue. Participants were told that they would have to find the big circle and then report the number of the abacus. The bead that contained the target item – i.e., the big circle – was then identified with a white ring, and participants were asked to indicate its color by pressing the appropriate "red" or "blue" button. After participants received feedback for their response, the abacus search array was removed from the screen, and participants were signaled to enter the number that had been depicted on the abacus, using their numeric keypad. Participants received 10 additional trials of a similar structure, in which the abacus represented a random 2 or 3 digit number, with the search target and distractors randomly distributed. For the last eight of these trials, search targets were not identified for the participants with white rings, but instead needed to be found. If participants responded incorrectly on any of the ten training trials – e.g., by incorrectly indicating the target's color or the abacus number – they began the ten trial loop again. All participants successfully completed the training phase.

Test phase. On each of 128 trials, participants viewed either a two- or three-column abacus schematic that depicted a number between 1 and 999 (see Figure 2).[4] As in training, participants were instructed to find the big circle and respond whether it was red or blue, while ignoring the big square and small circle distractor items (which were randomly assigned to be

[4] Single digit numbers (e.g., '1', '9', etc.) were represented with two abacus columns, using a leading zero (e.g., '01', '09').

blue or red). Reaction time and accuracy were recorded once the participant indicated the color of the target; after this, the participant entered the number represented by the abacus and we provided feedback and recorded their accuracy. As described below, we manipulated the location of the target bead to explore how MA experts allocate visual attention between beads in leading vs. trailing zero columns, and between in-play and out-of-play beads (see Figure 2).

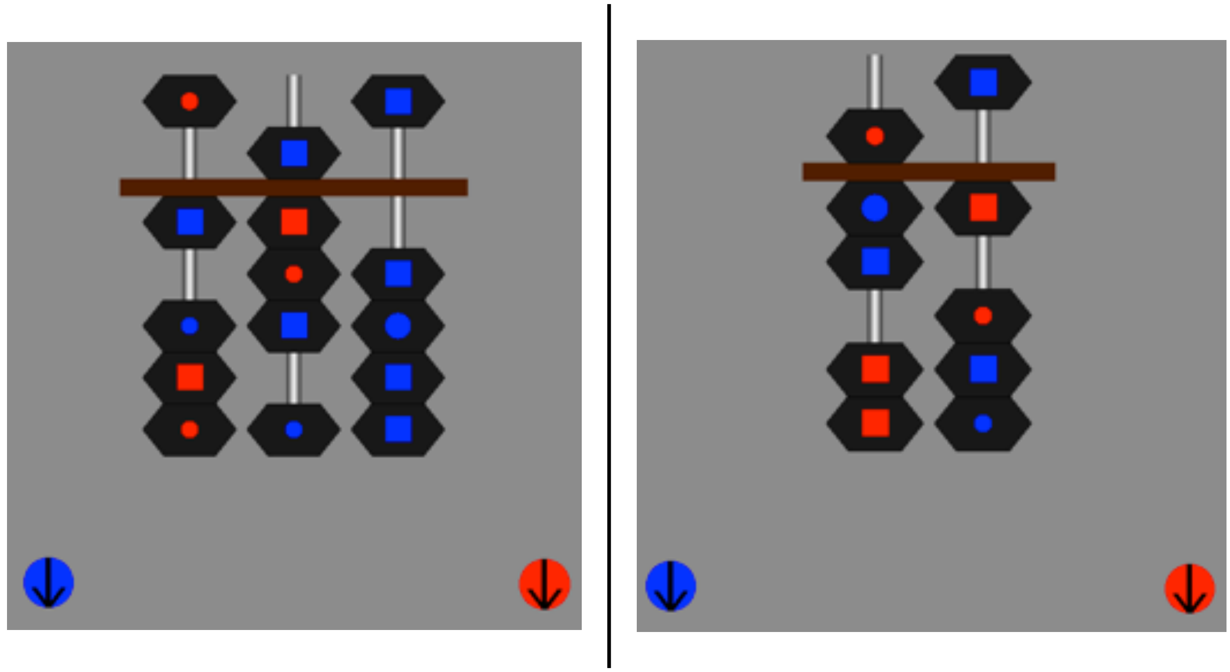

Figure 2. Examples of trials from the search tasks of Experiments 1 through 3. Left Panel: Example of a three-column trial in which the big circle (blue) is out-of-play and in a column representing a trailing zero. Right Panel: Example of a two-column trial in which the big circle is in-play.

Comparison of in-play vs out-of-play beads: In 64 of the trials, the target bead appeared in a randomly-chosen column that represented a digit between 0 and 9. Across these trials, the target bead appeared equally often in in-play vs. out-of-play positions, in two- vs. three-column abacus schematics, and was also equally likely to appear in each of the rows of beads. Because that abacus consists of one heavenly row of beads and four earthly rows of beads, this means that the target was four times as likely to appear on an earthly bead as on a heavenly bead.

Comparison of beads in leading vs trailing zero columns: In the other 64 trials, the target bead appeared in a column that represented either a leading or trailing zero, appearing in each of these columns an equal number of times, and equally often in two- or three-column abacus schematics. As in the in-play vs. out-of-play condition above, the target was also equally likely to appear in each of the rows. Although the beads in both leading and trailing zero columns are also necessarily out-of-play, we did not include trials in which targets appeared in these columns in the in-play vs out-of-play analyses described below.

Results and Discussion

To begin, participants correctly reported the number represented in the abacus displays on 95% of trials, indicating that they were engaged in reading these displays, as per our instructions. Thus, we have reason to believe that during the search task, participants were allocating their attention toward aspects of the abacus displays that are relevant to extracting number.

Our analyses for the visual search task were restricted to trials in which participants correctly indicated the color of the target, which occurred on 98% of trials. Reaction times were log transformed prior to analyses. Following standard practice, in all experiments reported here, all trials with log-transformed reaction times more than three standard deviations away from the mean were excluded from analysis. In the final dataset, participants took on average 3.76 seconds (95% CI: 3.56 – 3.98) to indicate the color of the target. We used a linear mixed-effects regression model (R version 3.3.0, lme4 version 1.1-12) to predict whether reaction times were affected by the position of target beads: i.e., whether they appeared on in-play beads, out-of-play beads, in leading zero columns, or in trailing zero columns, treating position on in-play beads as the intercept. Additionally, our model tested for whether reaction times were affected by the trial number of the task (to test for practice effects) and whether the abacus display had two or three columns; we also tested for interactions between the number of abacus columns and position of the target bead. We used the maximal convergent random effects structure (a random slope of condition and a random intercept for each participant; following Barr et al., 2013). All p-values were computed using the lmerTest package version . Raw data and full reports of analyses for all experiments reported in this paper can be found at: https://github.com/langcog/abacus_attn.

Our analyses revealed that subjects were faster to detect targets on in-play beads (M=3.63, 95% CI: 3.39-3.87) than on out-of-play beads (M=3.88, 3.67-4.12; β=.120, SE=.019, p<.001; Figure 3), consistent with the idea that MA experts preferentially deploy their attention toward in-play beads relative to out-of-play beads while reading the abacus. Further, while participants' speed to detect targets in trailing zero columns (M=3.66, 3.43-3.88) was similar to their speed to detect in-play targets, they were significantly slower to detect targets on beads in leading zero columns (M=3.86, 3.64-4.10; β=.154, SE=.018, p<.001; Figure 4), suggesting an attentional advantage for beads in trailing zero columns relative to beads in leading zero columns. Together, these results indicate that MA experts preferentially deploy their attention toward in-play beads and beads in trailing zero columns to extract the number represented by the abacus.

Also, our analyses indicated that participants were slower to detect targets when the abacus display had three as opposed to two columns (β=.253, SE=.018, p<.001), which is unsurprising given that the three-column displays had more distractors. Interestingly, however, we also found significant interactions between number of abacus columns and target bead position (all *ps* < .05). Specifically, the attentional advantage for in-play and trailing zero targets were stronger in the two-column trials. We return to this finding when discussing the results of Experiment 2.

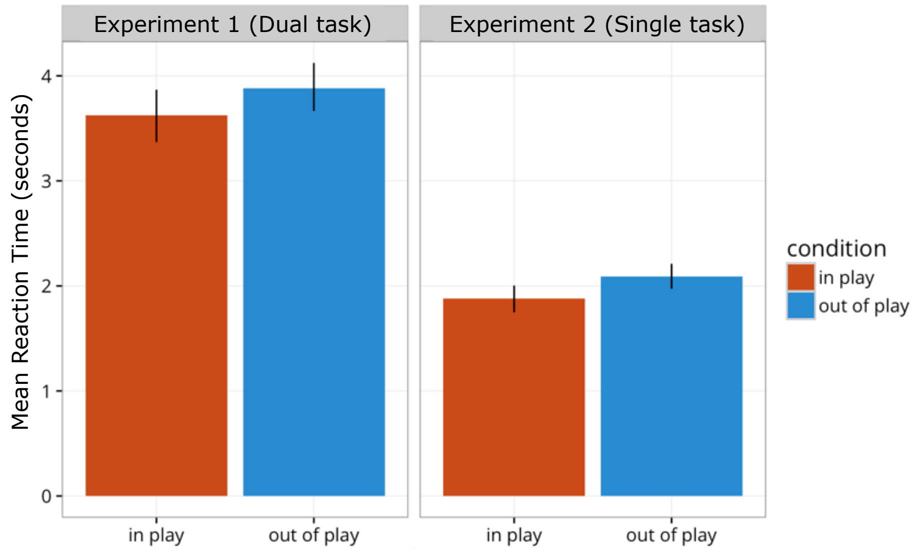

Figure 3. Average reaction time to detect targets on beads that were in-play vs. out-of-play in Experiments 1 and 2 (Error bars indicate 95% CI).

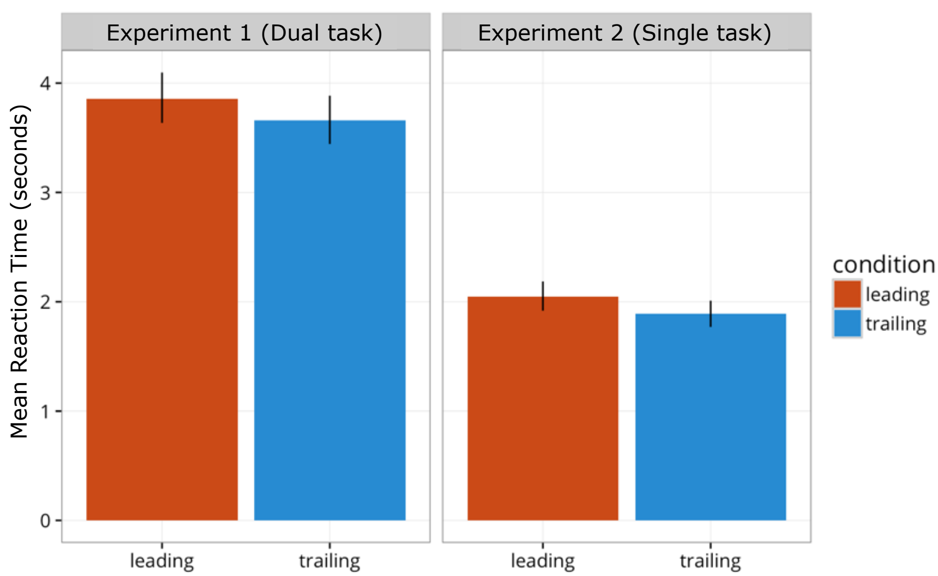

Figure 4. Average reaction time to detect targets on beads that were leading vs. trailing zero columns in Experiments 1 and 2 (Error bars indicate 95% CI).

Experiment 2

Experiment 1 suggested that, when asked to read the abacus, MA experts are faster to detect search targets on in-play beads (than out-of-play beads) and on beads in columns representing trailing zeroes (than on beads in columns representing leading zeroes), suggesting that they preferentially allocate their attention toward these aspects of the abacus while processing its number. In Experiment 2, we explored whether the biases observed toward in-play beads and beads in trailing zero columns only emerge when experts are explicitly asked to read the abacus, or if instead these biases may be more automatic and arise even when the intention is not to read the abacus. As evidence for the latter possibility, recall that, in a stroop-like paradigm, MA experts are slower to decide which of two abaci have a larger number of beads when the abacus with the larger number of beads represents a smaller numerical value (Du et al., 2014). This suggests that MA experts spontaneously read the number represented by an abacus display, even when they are not instructed to do so. To test whether the attentional biases observed in Experiment 1 might emerge automatically, in Experiment 2 we asked MA experts to complete only the visual search task: i.e., we did not also ask them to report the number represented by the abacus.

Method

Participants. The participants were 67 MA students, who had a mean age of 10.44 years (range: 7.47 to 14.36). MA students were included in our subject pool as described in the Participants section of Experiment 1.

Materials and Procedure. The methods and procedures were identical to those of Experiment 1 except that participants were not also asked to report the number represented by each abacus display, nor were they given training on this task.

Results and Discussion

As in Experiment 1, we restricted our analyses to trials in which participants correctly indicated the color of the target, which occurred on 96% of trials. On average, in included trials, participants took 1.97 seconds (1.85-2.09) to indicate the color of the target. As before, we used a mixed-effects model to predict whether reaction times were affected by the position of target beads (in-play, out-of-play, leading, or trailing), trial number, the number of abacus columns, and interactions between number of abacus columns and target bead position. This model used the same random effects structure as in Experiment 1: a random slope and intercept for each participant.

Consistent with the findings of Experiment 1, we found that subjects were faster to detect targets on in-play beads (M=1.88, 1.75-2.00) than on out-of-play beads (M=2.09, 1.97-2.21; β=.119, SE=.020, p<.001; Figure 3). Further, while participants' speed to detect targets in trailing zero columns (M=1.89, 1.77-2.02) did not differ from their speed to detect in-play targets, they were significantly slower to detect targets on beads in leading zero columns (M=2.05, 1.92-2.18; β=.092, SE=.020, p<.001; Figure 4). Finally, as in Experiment 1, participants were slower to detect targets in three-column than in two-column abacus displays (as in Experiment 1; β=.163, SE=.019, p<.001); however, we did not find significant interactions between number of abacus columns and target position (all ps>.54), indicating that attentional advantages for in-play and trailing zero targets were equally strong across the two- and three-column trials. This contrasts with Experiment 1, in which in-play and trailing-zero effects were weaker in three-column than in two-column trials; This raises the possibility that the load imposed by having to read three-column abaci in Experiment 1 may have suppressed attentional effects.

To directly assess the similarity of search patterns among experts who had been asked to read the abacus (Experiment 1) and those who had not (Experiment 2) we used another mixed-effects model to predict all participants' reaction times from Experiments 1 and 2. This model tested the effect of experiment, effects of target position, and interactions of experiment with target position. Interestingly, although participants were unsurprisingly faster to detect the target in Experiment 2 than in Experiment 1 (which required them to perform two concurrent tasks; β=-.797; *SE*=.049; *p*<.001), the effects of target position did not interact with experiment (all *p*s>.21), suggesting that subjects in the two experiments exhibited similar search patterns and attentional biases. Taken together, these results suggest that, even when they are not explicitly instructed to read the abacus, MA experts automatically deploy their attention toward semantically-relevant aspects of abacus structure, like in-play beads and beads in trailing zero columns.

Experiment 3

Experiment 3 explored how the automatic attentional biases documented in MA experts in Experiments 1 and 2 relate to the extensive training those experts have received. On one hand, practice using the abacus and knowledge of how the abacus represents number could create the visual attentional biases observed in MA experts, consistent with previous case studies showing that practice transforms the domain-relevant cognitive and perceptual processes that give rise to expertise (Chase & Simon, 1973; Chi, Glaser & Farr, 1988; Chi, Glaser & Rees, 1982; Ericsson & Lehman, 1996; Gauthier, Skudlarski, Gore & Anderson, 2000; McCormack et al., 2014; Sheridan & Reingold, 2007). By this account, extensive training could lead MA users to preferentially allocate their attention toward semantically-relevant aspects of abacus structure, like in-play beads, and beads in columns representing trailing zeroes. This account would then predict that naïve participants who do not know how the abacus represents number should fail to exhibit equivalent attentional biases, and might instead allocate their attention evenly across the abacus. On the other hand, abacus training may not shape visual attentional processes, but might instead be scaffolded from basic properties of visual attention. In particular, the biases observed in experts may not require extensive practice using the abacus, but may instead be present in experts and non-experts alike, and stem from the design and structure of the abacus itself. If so, then even naïve subjects may preferentially attend to semantically-relevant aspects of the abacus, facilitating the efforts of naïve users to learn MA.

To explore these questions, we conducted the visual search task of Experiments 1 and 2 with a group of adult participants from an American university community, and probed their familiarity with the abacus after they performed the task. This method allowed us to address whether biases toward in-play beads and beads in trailing zero columns are present in naïve subjects with no familiarity with the abacus, and if not, what levels of familiarity with the abacus might be required to develop such biases.

Method

Participants. Fifty-six adult participants (mean age = 21.24; range 18.42 – 33.27) were recruited from a participant database at the University of California, San Diego. All participants received course credit for their participation.

Materials and Procedure. All participants completed the visual search task prior to answering questions about their familiarity with the abacus.

Visual Search Task. The materials and procedures for the visual search task were the same as in Experiments 1 and 2.

Abacus Familiarity Assessment. After they completed the visual search task, we asked participants two questions to determine their level of familiarity with the abacus. First, we probed whether participants recognized that the display from the Visual Search Task represented an abacus: *Do you know what the display was?* Next, after telling them that the display depicted an abacus (if they had not said so themselves), we asked participants to describe their experience using an abacus: *Have you ever used an abacus*, *and if so, for how long?*

Based on their answers to these questions, we classified participants into three categories: “naïve”, “low familiarity” and “moderate familiarity”. Participants were considered “naïve” with respect to the abacus if they answered *no* to both questions (n = 32). They were considered to have “low familiarity” with the abacus (n = 14) if they stated that the display was an abacus (in addition to “abacus”, we accepted responses such as “counting beads” and “old calculator”) or reported using an abacus in the past despite failing to initially recognize that the display was an abacus. Finally, participants were considered to have “moderate familiarity” with the abacus (n = 10) if they *both* recognized that the display was an abacus *and* reported having used an abacus in the past. In general, even participants in the “moderate familiarity” group reported only sparse levels of prior experience with the abacus: e.g., four out of the seven participants in this group reported using the abacus only briefly as young children.

Results and Discussion

As before, we restricted our analyses to trials in which participants correctly indicated the color of the target, which occurred on 98% of trials. On included trials, participants took on average 1.20 seconds (1.12-1.27) to indicate the color of the target. We again used a mixed-effects model with the same random effects structure to predict whether reaction times were affected by the position of target beads (in-play, out-of-play, leading, or trailing), trial number, familiarity with the abacus (naïve, low, moderate), and by interactions between abacus familiarity and target bead position.[5]

Just as in Experiments 1 and 2 with MA experts, participants were faster to detect targets on in-play beads (*M*=1.15, 1.07-1.25) than on out-of-play beads (*M*=1.23, 1.15-1.30; β=.077, *SE*=.017, $p<.001$; Figure 5). Also as before, while participants' speed to detect targets in trailing zero columns (*M*=1.16, 1.10-1.24) did not differ from their speed to detect in-play targets, they were significantly slower to detect targets on beads in leading zero columns (*M*=1.23, 1.16-1.32; β=.056, *SE*=.017, $p<.005$; Figure 6).

Interestingly, naïve subjects (who have never used an abacus and cannot recognize one) showed just as strong of an advantage for in-play beads as did subjects in the low and moderate familiarity groups (Figure 5): There were no significant interactions between levels of abacus familiarity and position on out-of-play (as opposed to in-play) beads (all *p*s > .37). In contrast, there was a significant interaction between abacus familiarity and target position in leading vs. trailing columns, as subjects in the moderate familiarity group exhibited a greater trailing vs. leading effect, compared to naive subjects (β=.116, *SE*=.044, $p=.010$). To test whether even the naïve subjects (N=32) showed the leading zero effect, we fit a follow-up model to the data from only these participants and again found a significant leading zero effect (β=.056, *SE*=.015, $p<.001$; Figure 6).

[5] A preliminary model that included number of abacus columns did not yield any significant interactions with effects of target position (as in Experiment 2); This factor was excluded from subsequent models.

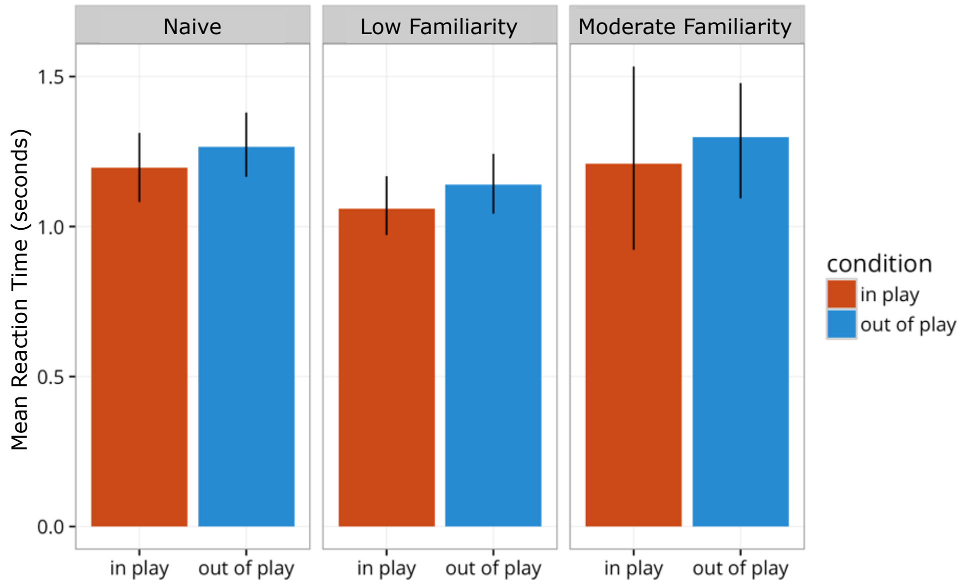

Figure 5. Average reaction time to detect targets on beads that were in-play vs. out-of-play by subjects in the naïve, low familiarity, and moderate familiarity groups of Experiment 3 (Error bars indicate 95% CI).

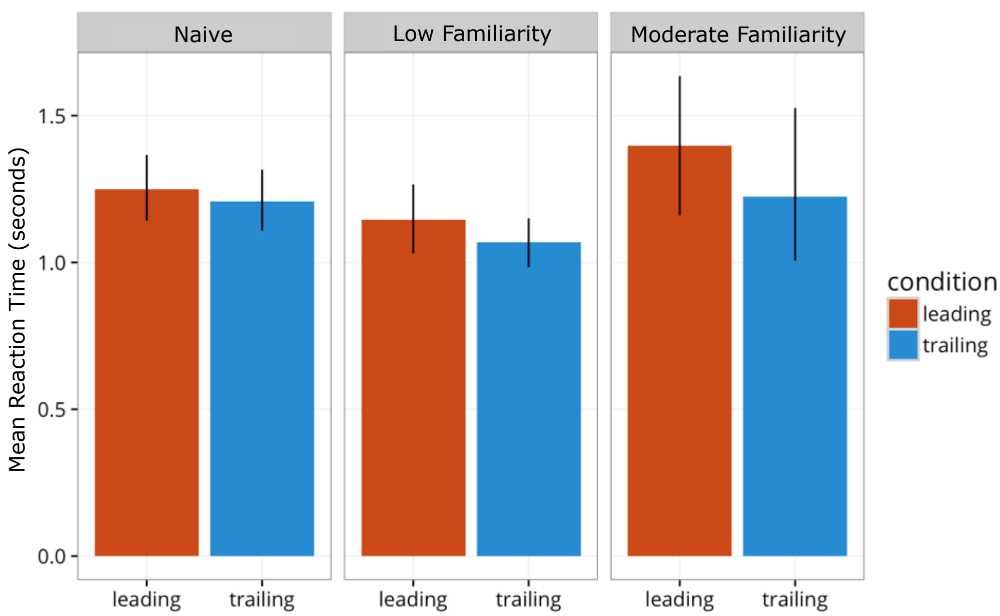

Figure 6. Average reaction time to detect targets on beads in leading vs. trailing zero columns by subjects in the naïve, low familiarity, and moderate familiarity groups of Experiment 3 (Error bars indicate 95% CI).

To directly assess the similarity of search patterns between the subjects of Experiment 3 – who had little to no experience using the abacus – and the MA experts from Experiment 2, we used a final mixed-effects model. This model tested the effect of experiment, effects of target position, and interactions of experiment with target position. Although participants from Experiment 3 were on average faster to detect the target than MA experts in Experiment 2 (β=-.450; *SE*=.047; *p*<.001),[6] the effects of target position did not interact with experiment (all *p*s>.05), suggesting that subjects in the two experiments exhibited similar search patterns and attentional biases. Taken together, these results suggest that the automatic attentional biases that scaffold numerical processing in abacus experts are present in naïve subjects.

Post-hoc Analyses of the In-play Bias

Thus far, we have explored how MA experts (Experiments 1 and 2) and participants with little to no familiarity with the abacus (Experiment 3) allocate their attention toward the abacus. Our evidence suggests that a bias to attend toward in-play beads relative to out-of-play beads is present in entirely naïve participants, as is a bias to attend toward trailing zeroes relative to leading zeroes (though only the former is equally strong among entirely naïve subjects and subjects with more familiarity with the abacus; Experiment 3). This pattern of findings suggests that in some cases, the design of the abacus takes advantage of general attentional biases we all share to direct attention toward semantically-relevant aspects of abacus structure. Here, in a series of post-hoc analyses, we explored what more general attentional biases might underlie the advantage for in-play beads, and if the same biases guide both expert MA users and participants with little to no experience with the abacus.

[6] This finding could indicate that MA experts – but not subjects with little to no familiarity with the abacus – automatically process the abacus prior to searching for the target. However, the slower speed of MA experts in Experiment 2 could also be explained by the fact that those subjects were children, while the participants in Experiment 3 were adults.

To begin, we explored whether the advantage for in-play beads could be explained by a general attentional bias that favors targets that are closer to the horizontal beam over those that are further away. In particular, attention may be drawn first to the horizontal beam due to its relative salience (e.g., it has a distinct thickness and color, and contrasts in orientation with vertical columns of beads; see Koch & Ullman, 1987; see also Itti & Koch, 2001; Itti et al., 1998; Li et al., 2002; Treisman & Gelade, 1980), such that locations further from the beam are less selected for attention than locations closer to the beam. This would explain the in-play advantage, because in-play beads are generally closer to the horizontal beam than out-of-play beads. But critically, if this is correct, then in some cases out-of-play targets should be detected *faster* than in-play targets due to the in-play beads being further away from the beam, i.e., in cases where several earthly beads within a column are in-play, and the target in-play bead does not directly adjoin the beam.

To test the prediction that proximity to the beam explains the in-play advantage over and above in-play/out-of-play status, we used mixed-effect models to explore whether subjects' speed to detect the target in each experiment was affected by in-play/out-of-play status (we excluded leading / trailing trials from analysis), trial number, proximity of the target to the beam, and an interaction between in-play/out-of-play status and proximity to the beam. This analysis was confined to earthly beads, since in-play status and proximity to the beam are confounded for heavenly beads. Consistent with the idea that the in-play advantage can be explained by a bias to attend toward targets that are closer to the horizontal beam, our models for Experiments 1 and 3 detected effects of proximity to the beam (Experiment 1: $\beta=.030$, $SE=.011$, $p=.005$; Experiment 3: $\beta=.044$, $SE=.009$, $p<.001$) but no effect of in-play/out-of-play status (Experiment 1: $\beta=-.005$, $SE=.077$, $p=.94$; Experiment 3: $\beta=.090$, $SE=.069$, $p=.19$), or interactions between in-play/out-of-play status and proximity to the beam (Experiment 1: β=.005, *SE*=.015, *p*=.75; Experiment 3: β=.009, *SE*=.013, *p*=.48). These findings suggest that, for the MA experts in Experiment 1 (who performed the dual search-reading task), and the naïve subjects in Experiment 3 (who performed only the search task), beads that were closer to the beam were more likely to be selected for attention, suggesting that the in-play advantage emerges from this general attentional bias.

Interestingly, however, our model of the search patterns of MA experts in Experiment 2 (who performed only the search task, just like the mostly naïve subjects of Experiment 3 described above) yielded a different pattern of effects. Here, proximity to the beam did not emerge as a significant predictor (β=.004, *SE*=.011, *p*=.71), though in-play/out-of-play status did (β=-.279, *SE*=.084, *p*=.001). The in-play effect was also qualified by an interaction between in-play/out-of-play status and proximity to the beam (β=.059, *SE*=.016, *p*<.001), such that participants were slower to detect out-of-play targets – but not in-play targets – that were further away from the beam. The fact that proximity to the beam did not affect MA experts' speed to detect targets on in-play beads in Experiment 2 suggests that they automatically allocated their attention across these beads, and perhaps treated them as a "single object" of attention (for review, see Chen, 2012; Scholl, 2001; see also Duncan, 1984; Egly, Driver & Rafal, 1994; Luck & Vogel, 1997; Treisman, 1982; Vecera & Farah, 1994).

In sum, our post-hoc analyses provide some evidence that the in-play advantage observed in MA experts in Experiment 1 and entirely naïve subjects in Experiment 3 might be explained by a more general attentional bias to attend toward salient objects (e.g., the horizontal cross-beam), such that other proximal objects are also likely to be selected for attention (e.g., in-play or out-of-play beads that are close to the beam), since effects of proximity to the beam emerged over and above in-play/out-of-play status in both experiments. However, the fact that MA experts from Experiment 2 – who performed an equivalent task to the naïve subjects from Experiment 3 – did not show an effect of proximity to the beam on in-play targets suggests another possibility: As a consequence of training, MA experts may automatically allocate their attention across in-play beads, by selecting these beads as a single "object" of attention.

General Discussion

The present studies explored whether the abacus – a modern descendent of the first human computing devices – evolved to exploit the constraints of human visual attention, or whether, instead, abacus expertise involves extending or adapting the capacity of human visual attention through practice. To address this question, we administered a series of visual search tasks to MA experts and subjects who had little to no experience with the abacus. In these tasks, search targets and distractors were overlaid atop abacus "beads." Using this method, Experiment 1 found that when asked to read an abacus, MA experts were faster to detect targets that were in-play, and targets in columns representing trailing zeroes, providing evidence that these aspects of abacus structure are relevant to extracting abacus number. Experiment 2 built upon these results and found that, for MA experts, attentional biases toward in-play and trailing zero beads may be automatic, as they emerge even when experts are not explicitly asked to read the abacus. Finally, in Experiment 3 we found that, like MA experts, subjects with no experience or familiarity with the abacus also show an advantage for in-play beads, and for beads in trailing zero columns (though the latter tendency is stronger when subjects have more familiarity with the abacus). Together, these findings suggest that the automatic biases that scaffold numerical processing in MA experts require little to no experience with the abacus to develop, and thus emerge from general properties of visual attention that are exploited by the design of the abacus itself.

By suggesting that the development of MA expertise is in part facilitated by the design of the abacus, our results complement previous accounts of the development of expertise, which have argued that expertise may stem from individual differences (Gobet & Ereku, 2007; Smith, Tsimpli & Ouhalla, 1993), or require extensive, deliberate practice (Charness, Krampe & Mayr, 1996; Ericsson, Krampe & Tesch-Romer, 1993; Ericsson & Lehman, 1996; Platz et al., 2014; Starkes et al., 1996) to transform domain-relevant representations and processes (Chase & Simon, 1973; Chi, Glaser & Farr, 1988; Chi, Glaser & Rees, 1982; Ericsson & Lehman, 1996; Gauthier, Skudlarski, Gore & Anderson, 2000; McCormack, 2014; Sheridan & Rheingold, 2014). Consequently, our results have implications for how easily and widely MA expertise can be attained, and thus its educational utility. In particular, by encouraging even novices to preferentially attend to numerically-relevant aspects of the abacus, like in-play beads and beads in trailing zero columns (Experiment 3), the design of the abacus may scaffold learning how the abacus represents number, and thus make MA expertise more easily attainable. This feature of the abacus may hold broader lessons for the use of concrete manipulatives in mathematical education (Ball, 1992; Uttal, Scudder, & Deloache, 1997), as it suggests that such manipulatives will be most effective when their design takes into account the perceptual and cognitive abilities of their users.

What general aspects of visual attention might give rise to the advantage for in-play beads observed in both MA experts and naïve subjects? Our post-hoc analyses suggest that one critical factor in explaining the in-play advantage is the fact that beads that are in-play tend to be closer to the horizontal beam than beads that are out-of-play. Indeed, we found that when proximity to the beam was entered in as a predictor of reaction times, the in-play advantage vanished for MA experts in Experiment 1 and naïve subjects in Experiment 3, suggesting that the in-play advantage observed in these experiments reduced to a bias to attend toward locations that are closer to the horizontal beam. To explain this, we have suggested that attention may be initially drawn to the beam because of its unique thickness, color, and orientation – e.g., early visual processes could identify the beam as the most salient region of the visual scene (see Koch & Ullman, 1987; see also Itti & Koch, 2001; Itti et al., 1998; Li et al., 2002; Treisman & Gelade, 1980) – making other neighboring areas, like in-play beads, more likely to be selected for attention. One intriguing possibility left open by our data is whether attention extends automatically across all in-play beads for MA experts, because the beam and in-play beads are treated as a single "object" of attention (for review, see Chen, 2012; Scholl, 2001; see also Duncan, 1984; Egly, Driver & Rafal, 1994; Luck & Vogel, 1997; Treisman, 1982; Vecera & Farah, 1994).

Regardless of what biases in visual attention give rise to the in-play advantage, our results suggest that the design of the abacus capitalizes on such biases to direct attention toward semantically-relevant aspects of the abacus, like in-play beads. Is this simply a coincidence? One intriguing possibility is that the structure of the abacus evolved over time, and became optimally tailored to properties of visual attention and human cognition more generally (Frank & Barner, 2011). By this account, early versions of the abacus may not have been as easy to use or as powerful as current forms (see Ifrah, Harding, Bellos & Wood, 2000), and these perceived shortcomings may have driven further innovation and changes to the basic design of the abacus, as appears to be the case with the design of many artifacts (Basalla, 1988; Petroski, 1993).

Consistent with the idea that the design of the abacus could have evolved over time, the abacus has a long cultural history (Ifrah, Harding, Bellos & Wood, 2000; Menninger, 1969), as early variants were present in ancient Mesopotamia, Greece (the *Salamis Tablet*), Rome (the

Roman Hand Abacus), and China (the *Suan Pan*). Further, it is known that current versions of the abacus, including the specific abacus studied here – the Japanese *Soroban* – have been modified from their early forms, e.g., by reducing the number of beads in each column (Frederic, 2005). Such modifications may have resulted in design features that have made the abacus optimally tailored to properties of visual attention and human cognition more generally. For example, both the *Suan Pan* and the *Soroban* include a salient horizontal divider and vertically-oriented columns of beads, and these features could mediate how visual attention is allocated: The salience of the horizontal divider could attract attention to in-play beads, and the vertical grouping of columns of beads could encourage columns to be treated as "objects" in working memory. Although little is currently known about the degree to which psychological factors have shaped historical modifications in abacus design across cultures, it would seem logical to think that they have, since computations using the physical abacus are distributed (Hutchins, 1995; Zhang & Norman, 1994), and require coordination among the internal mental representations of abacus users and the external knowledge representations instantiated by the abacus.

More centrally, another reason to think that the abacus may have evolved to coordinate well with human perception and cognition is the fact that MA is possible at all – i.e., that individuals can perform computations using mental images of an abacus – and that MA expertise can be widely attained. If the abacus was difficult to hold in memory and process, MA would be difficult, and MA expertise might only be attained by those with unusually strong visuo-spatial resources. However, as reviewed in the Introduction, acquiring MA expertise does not require unusual visuo-spatial resources, such as augmented spatial working memory or mental rotation capacities; Instead, even children with typical levels of visuo-spatial ability can readily acquire MA expertise (Barner et al., 2016). Further, MA experts do not exhibit unusual perceptual expertise, but instead resemble naïve subjects. MA users are not faster or more accurate at estimating the cardinality of dot arrays than naïve subjects (Frank & Barner, 2011); Instead, both naïve subjects and MA experts are better at numerical estimation when dot arrays are configured similarly to the abacus (Frank & Barner, 2011). In sum, by showing that the attentional biases exhibited by MA experts are shared by novices, the present studies complement previous findings, and suggest that the abacus may have evolved to make optimal use of pre-existing visuo-spatial resources.

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